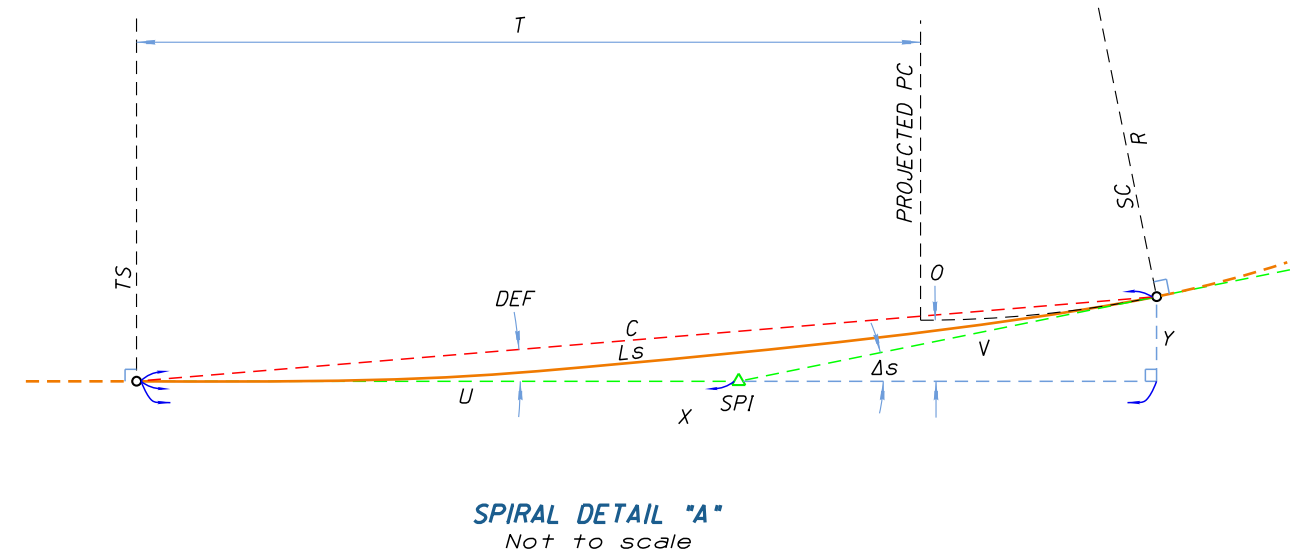
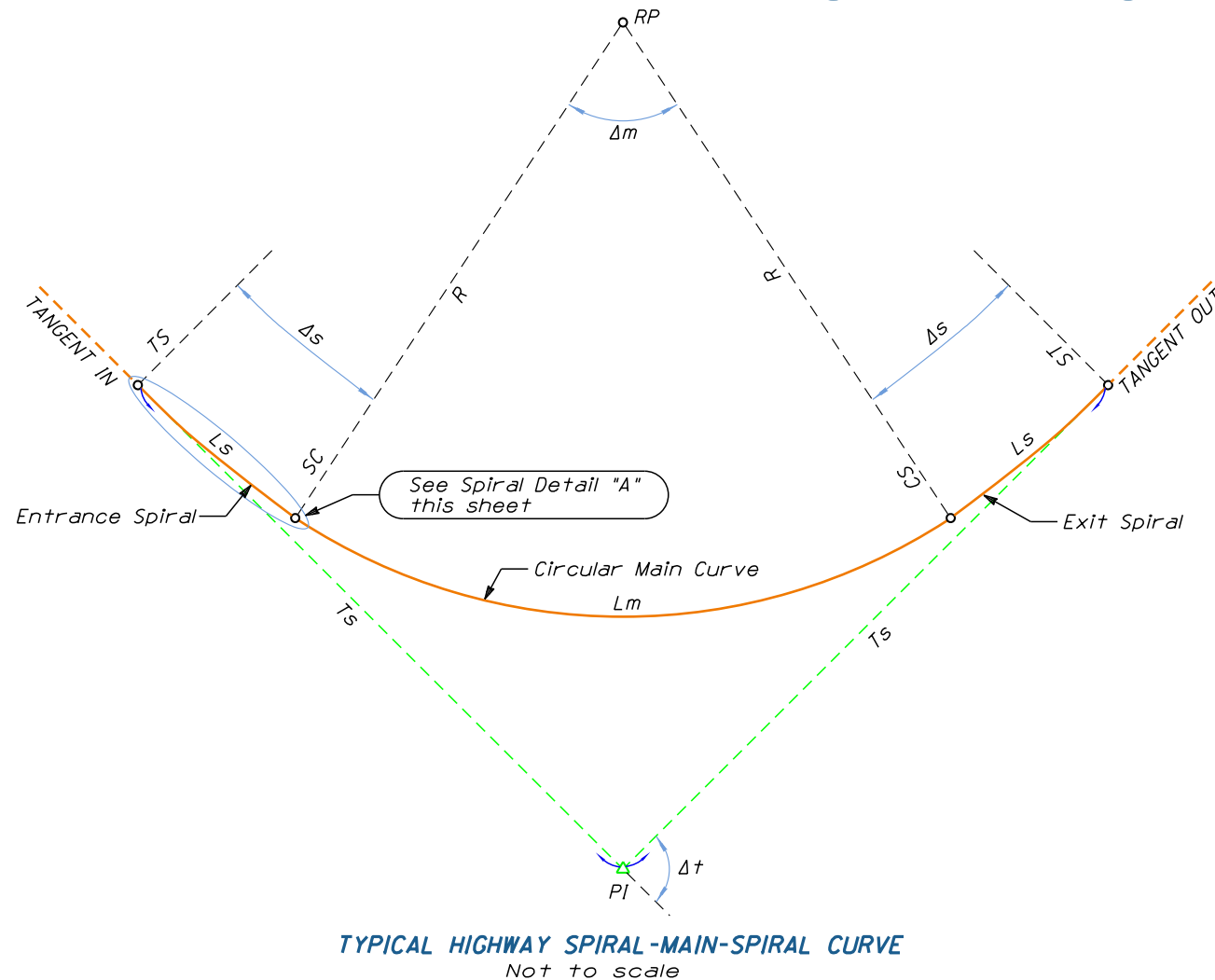


FULL TRANSITION SPIRAL CURVE



Highway Spiral Curve where $\Delta s < 16^\circ$

FORMULAS

$$\begin{aligned} R &= 5729.57795 / D \\ a &= (D * 100) / Ls \\ O &= 0.0727 * a * ((Ls / 100)^3) \\ T &= (Ls / 2) - (0.000127 * a^2 * (Ls / 100)^5) \\ Ts &= (Tan(\Delta t / 2) * (R + O)) + T \\ C &= Ls - (0.00034 * a^2 * (Ls / 100)^5) \\ DEF &= (a * Ls^2) / 60000 \\ \Delta s &= 0.005 * D * Ls \\ U &= C * Sin(\Delta s * 2 / 3) / Sin(\Delta s) \\ V &= C * Sin(\Delta s * 1 / 3) / Sin(\Delta s) \\ \Delta m &= \Delta t - \Delta s - \Delta s \\ Lm &= (\Delta m * R * \pi) / 180 \\ X &= C * Cos(DEF) \\ Y &= C * Sin(DEF) \end{aligned}$$

$$\begin{aligned} SC(S+a) &= TS(S+a) + Ls \\ CS(S+a) &= SC(S+a) + Lm \\ ST(S+a) &= CS(S+a) + Ls \\ PI(S+a) &= TS(S+a) + Ts \end{aligned}$$

EXAMPLE

Given: $\Delta t = 36^\circ 29' 16''$; $D = 2^\circ 00' 00''$; $L_s = 200'$; $TS(S+a) = 2180+84.70$

$$\begin{aligned}
 R &= 5729.57795 / 2.0000^\circ = \underline{2864.78898} \\
 a &= (2.0000^\circ * 100) / 200 = \underline{1.00} \\
 O &= 0.0727 * 1 * ((200 / 100)^3) = \underline{0.58160} \\
 T &= (200 / 2) - (0.000127 * 1^2 * (200 / 100)^5) = \underline{99.99594} \\
 Ts &= (\tan(36.48777777^\circ / 2) * (2864.78898 + 0.58160)) + 99.99594 = \underline{1044.51462} \\
 C &= 200 - (0.00034 * 1^2 * (200 / 100)^5) = \underline{199.98912} \\
 DEF &= (1 * 200^2) / 60000 = \underline{0.666667^\circ \text{ or } 0^\circ 40' 00''} \\
 \Delta s &= 0.005 * 2.0000^\circ * 200 = \underline{2.0000^\circ \text{ or } 2^\circ 00' 00''} \\
 U &= 199.98912 * \sin(2.0000^\circ * 2 / 3) / \sin(2.0000^\circ) = \underline{133.34112} \\
 V &= 199.98912 * \sin(2.0000^\circ * 1 / 3) / \sin(2.0000^\circ) = \underline{66.67508} \\
 \Delta m &= 36^\circ 29' 16'' - 2^\circ 00' 00'' - 2^\circ 00' 00'' = \underline{32^\circ 29' 16'' \text{ or } 32.48777777^\circ} \\
 Lm &= (32.48777777^\circ * 2864.78898 * 3.141592654) / 180 = \underline{1624.38889} \\
 X &= 199.98912 * \cos(0.666667^\circ) = \underline{199.97558} \\
 Y &= 199.98912 * \sin(0.666667^\circ) = \underline{2.32693}
 \end{aligned}$$

$$\begin{aligned} SC(S+a) &= 2180+84.70 + 200 = \underline{2182+84.70} \\ CS(S+a) &= 2182+84.70 + 1624.39 = \underline{2199+09.09} \\ ST(S+a) &= 2199+09.09 + 200 = \underline{2201+09.09} \\ PI(S+a) &= 2180+84.70 + 1044.51 = \underline{2191+29.21} \end{aligned}$$

DEFINITIONS

Δt = Total Delta Deflection
 D = Degree of Curvature
 L_s = Length of Spiral
 $TS(Sta)$ = Tangent to Spiral
 R = Radius
 a = Rate of change per 100'
 O = Radial Offset
 T = Projected curve P.C.
 T_s = Tangent Length
 C = Spiral Chord
 DEF = Deflection angle at $TS(Sta)$
 Δs = Spiral Delta
 U = Distance to Spiral PI
 V = Distance to $SC(Sta)$
 Δm = Main Curve - Delta
 L_m = Main Curve - Length
 X = Distance along X axis
 Y = Distance along Y axis
 $SC(Sta)$ = Spiral to Curve
 $CS(Sta)$ = Curve to Spiral
 $ST(Sta)$ = Spiral to Tangent
 $PI(Sta)$ = Point of Intersection
 RP = Radius Point of Main Curve
 SPI = Spiral Point of Intersection
 POS = Point on Spiral

FULL TRANSITION SPIRAL CURVE
EQUAL ENTRANCE & EXIT SPIRALS

Professional Land
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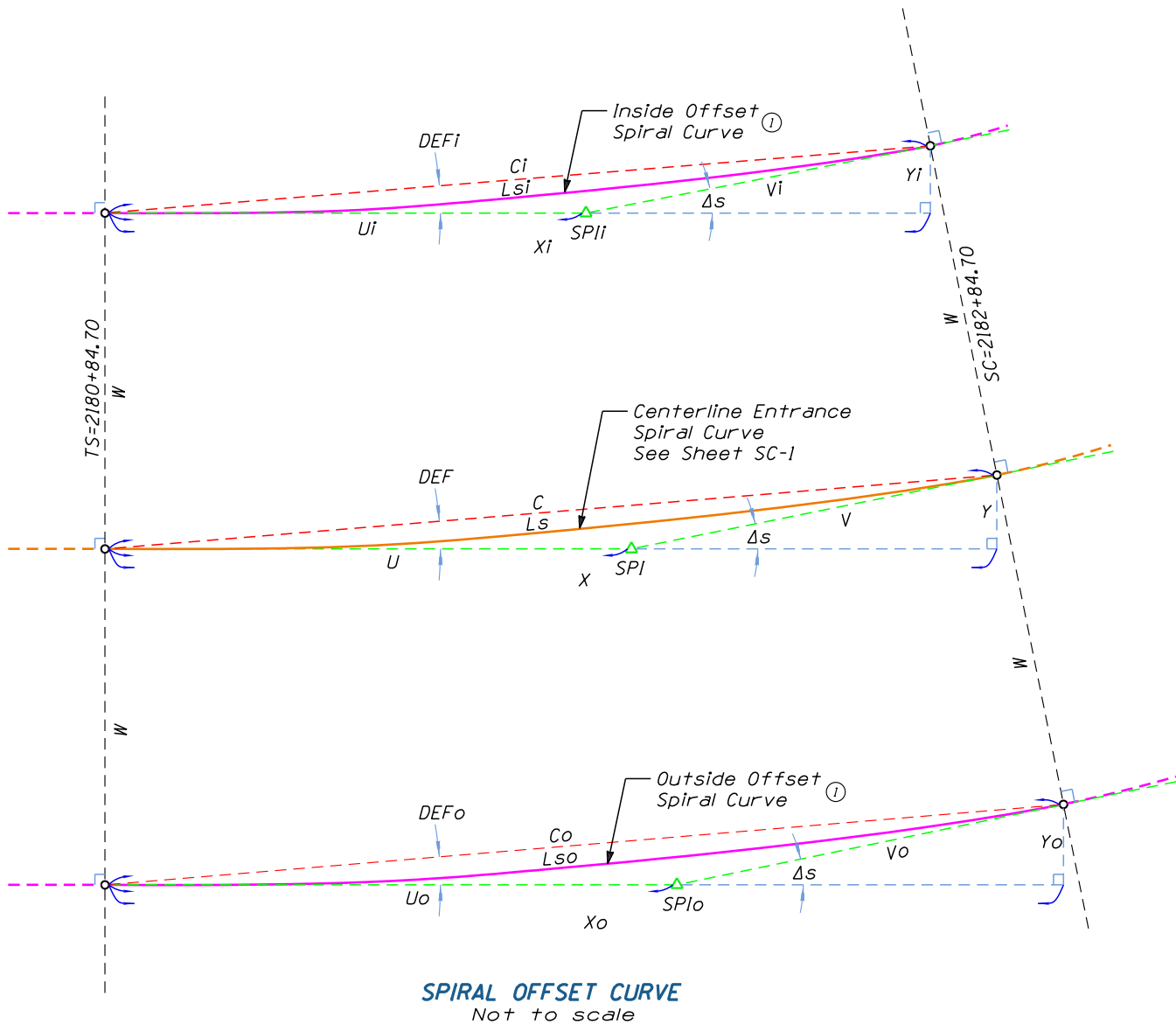
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Date	Jun 2011
Project Sur.	Jim Crume

Project No. ADOT Spiral	Project Manager Jim Crume
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SC-1
Sht: 1 of 7

SPIRAL OFFSET CURVE



DEFINITIONS

W = Offset distance
 Xi / Xo = Distance along X axis
 Yi / Yo = Distance along Y axis
 Ci / Co = Offset Spiral Chord
 Vi / Vo = Distance to SC(Sta)
 Ui / Uo = Distance to Offset Spiral PI
 DEFi / DEFo = Offset Spiral Deflection angle
 Lsi / Lso = Offset Spiral Length
 SPIi / SPIo = Offset Spiral PI

See SC-1 for additional definitions

① Offset Spiral Curve characteristics vary slightly from the centerline spiral. The formula's on this sheet are dependent upon the centerline spiral as described on sheet SC-1.

FORMULAS

INSIDE OFFSET SPIRAL

$$Xi = X - (\sin(\Delta s) * W)$$

$$Yi = Y - W + (\cos(\Delta s) * W)$$

$$Ci = \sqrt{Xi^2 + Yi^2}$$

$$Vi = Yi / \sin(\Delta s)$$

$$Ui = Xi - (Yi / \tan(\Delta s))$$

$$Lsi = Ci * Ls / C$$

$$DEFi = \arctan(Yi / Xi)$$

$$Ri = R - W$$

$$Di = 5729.57795 / Ri$$

$$ai = Di * 100 / Lsi$$

OUTSIDE OFFSET SPIRAL

$$Xo = X + (\sin(\Delta s) * W)$$

$$Yo = Y + W - (\cos(\Delta s) * W)$$

$$Co = \sqrt{Xo^2 + Yo^2}$$

$$Vo = Yo / \sin(\Delta s)$$

$$Uo = Xo - (Yo / \tan(\Delta s))$$

$$Lso = Co * Ls / C$$

$$DEFo = \arctan(Yo / Xo)$$

$$Ro = R + W$$

$$Do = 5729.57795 / Ro$$

$$ao = Do * 100 / Lso$$

Alternate formula's for Spiral Lengths as published in "Route Location and Design" by Thomas Felix Hickerson (1967)

$$Lsi = Ls - (0.017453 * W * \Delta s)$$
 and
$$Lso = Ls + (0.017453 * W * \Delta s)$$

EXAMPLE

Given: $\Delta s = 2^\circ 00' 00''$; $Ls = 200$; $C = 199.98912'$; $R = 2864.78898$ (From SC-1)
 $X = 199.97558$; $Y = 2.32693$; $W = 100'$

INSIDE OFFSET SPIRAL

$$Xi = 199.97558 - (\sin(2.0000^\circ) * 100) = 196.48563$$

$$Yi = 2.32693 - 100 + (\cos(2.0000^\circ) * 100) = 2.26601$$

$$Ci = \sqrt{196.48563^2 + 2.26601^2} = 196.49870$$

$$Vi = 2.26601 / \sin(2.0000^\circ) = 64.92959$$

$$Ui = 196.48563 - (2.26601 / \tan(2.0000^\circ)) = 131.59559$$

$$Lsi = 196.49870 * 200 / 199.98912 = 196.50939$$

$$DEFi = \arctan(2.26601 / 196.48563) = 0.66075^\circ \text{ or } 00^\circ 39' 39''$$

$$Ri = 2864.78898 - 100 = 2764.78898$$

$$Di = 5729.57795 / 2764.78898 = 2.07234^\circ \text{ or } 2^\circ 04' 20''$$

$$ai = 2.07234 * 100 / 196.50939 = 1.05458$$

OUTSIDE OFFSET SPIRAL

$$Xo = 199.97558 + (\sin(2.0000^\circ) * 100) = 203.46553$$

$$Yo = 2.32693 + 100 - (\cos(2.0000^\circ) * 100) = 2.38785$$

$$Co = \sqrt{203.46553^2 + 2.38785^2} = 203.47954$$

$$Vo = 2.38785 / \sin(2.0000^\circ) = 68.42076$$

$$Uo = 203.46553 - (2.38785 / \tan(2.0000^\circ)) = 135.08645$$

$$Lso = 203.47954 * 200 / 199.98912 = 203.49061$$

$$DEFo = \arctan(2.38785 / 203.46553) = 0.67239^\circ \text{ or } 00^\circ 40' 21''$$

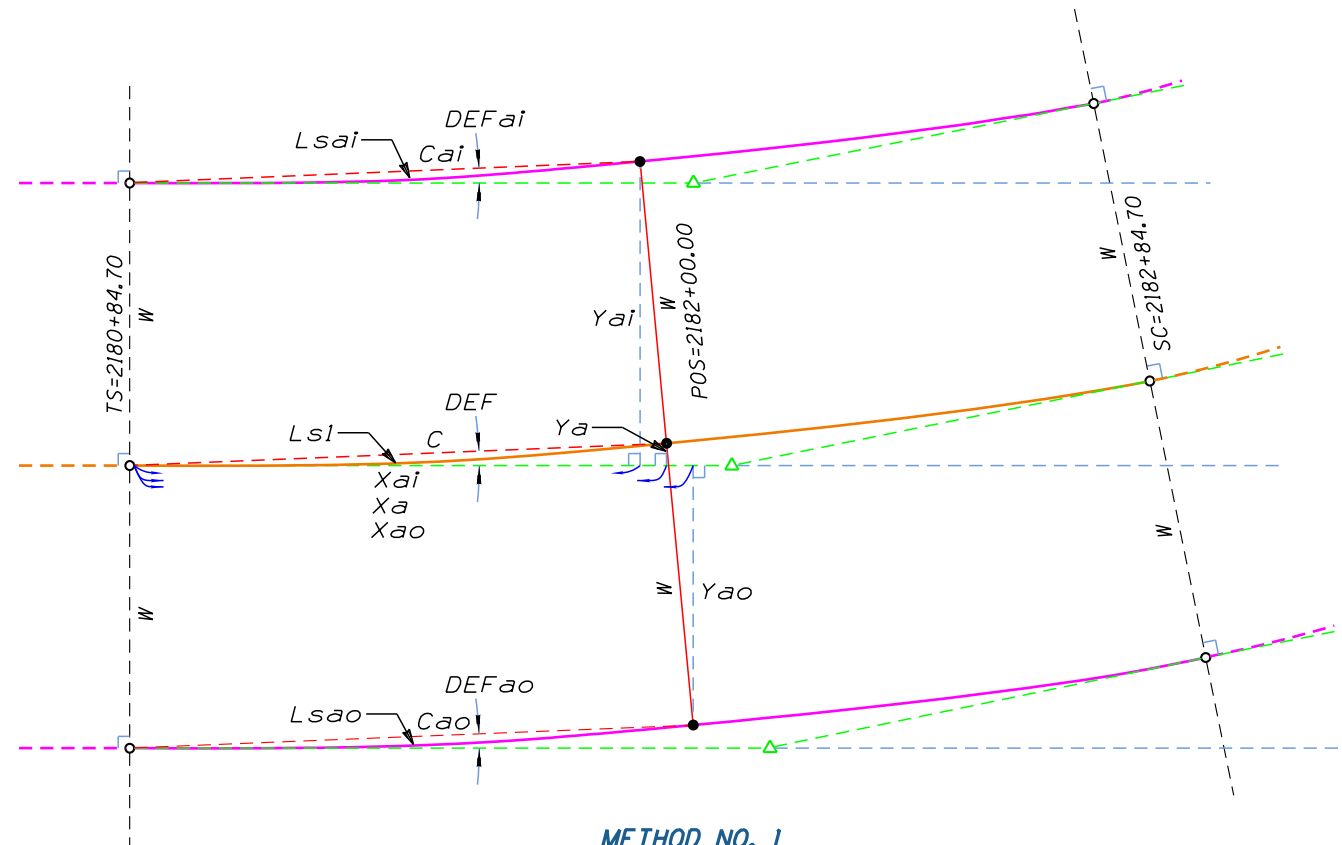
$$Ro = 2864.78898 + 100 = 2964.78898$$

$$Do = 5729.57795 / 2964.78898 = 1.93254^\circ \text{ or } 1^\circ 55' 57''$$

$$ao = 1.93254 * 100 / 203.49061 = 0.94969$$

SPIRAL OFFSET CURVE TABLE												
	W	a	D	R	Length	Delta	DEF	Chord	X	Y	U	V
INSIDE	100	1.05463	2°04'20"	2764.789	196.509	2°00'00"	00°39'39"	196.499	196.486	2.266	131.596	64.930
	50	1.02678	2°02'08"	2914.789	198.255	2°00'00"	00°39'49"	198.244	198.231	2.296	132.468	65.802
	0	1.00000	2°00'00"	2864.789	200.00	2°00'00"	00°40'00"	199.989	199.976	2.326	133.341	66.675
OUTSIDE	50	0.97440	1°57'57"	2914.789	201.745	2°00'00"	00°40'10"	201.734	201.721	2.357	134.214	67.548
	100	0.94975	1°55'57"	2964.789	203.491	2°00'00"	00°40'21"	203.480	203.466	2.388	135.087	68.421

POINTS ON SPIRAL OFFSET CURVE



METHOD NO. 1
Not to scale

FORMULAS

$$C = Lsl - (0.00034 * a^2 * (Lsl / 100)^5); \quad Lsl = POS - TS$$

$$DEF = (a * Lsl^2) / 60000$$

$$Xa = C * \cos(DEF)$$

$$Ya = C * \sin(DEF)$$

$$Xai = Xa - (\sin(DEF * 3) * W)$$

$$Yai = Ya + (\cos(DEF * 3) * W)$$

$$Xao = Xa + (\sin(DEF * 3) * W)$$

$$Yao = Ya - (\cos(DEF * 3) * W)$$

$$Cai = \sqrt{Xai^2 + (Yai - W)^2}$$

$$Lsai = Cai * Lsl / C$$

$$DEFai = \arctan((Yai - W) / Xai)$$

$$Cao = \sqrt{Xao^2 + (Yao + W)^2}$$

$$Lsao = Cao * Lsl / C$$

$$DEFao = \arctan((Yao + W) / Xao)$$

EXAMPLE

Given: $a = 1$ $TS = 2180+84.70$ (From SC-1) $POS = 2182+00.00$ $W = 100'$

$$Lsl = 2182+00.00 - 2180+84.70 = 115.30'$$

$$C = 115.30 - (0.00034 * 1^2 * (115.30 / 100)^5) = 115.29931$$

$$DEF = (1 * 115.30^2) / 60000 = 0.22157^\circ \text{ or } 0^\circ 13' 18''$$

$$Xa = 115.29931 * \cos(0.22157) = 115.29845$$

$$Ya = 115.29931 * \sin(0.22157) = 0.44588$$

$$Xai = 115.29845 - (\sin(0.22157 * 3) * 100) = 114.13834$$

$$Yai = 0.44588 + (\cos(0.22157 * 3) * 100) = 100.43915$$

$$Cai = \sqrt{114.13834^2 + (100.43915 - 100)^2} = 114.13918$$

$$Lsai = 114.13918 * 115.30 / 115.29931 = 114.13986$$

$$DEFai = \arctan((100.43915 - 100) / 114.13834) = 0.22045^\circ \text{ or } 0^\circ 13' 14''$$

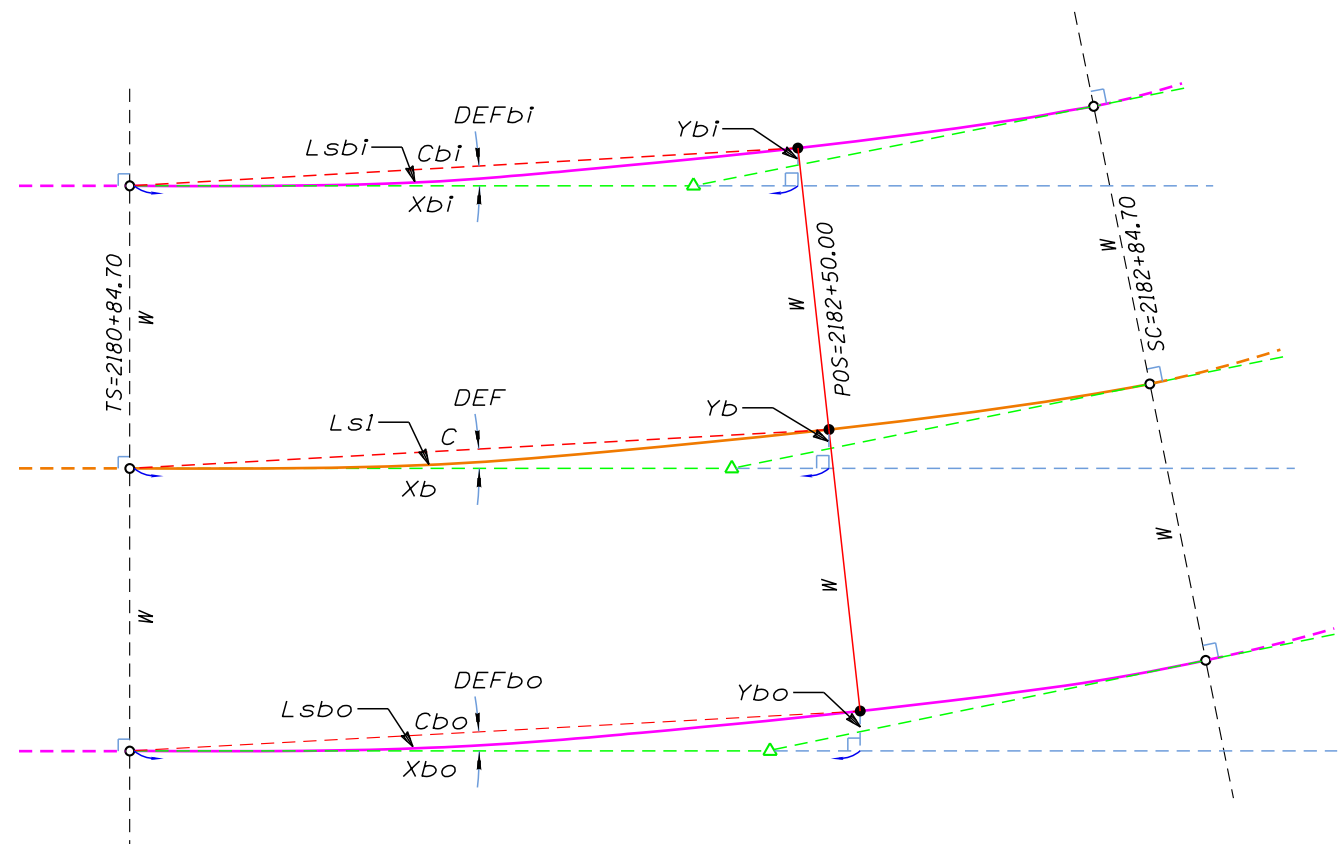
$$Xao = 115.29845 + (\sin(0.22157 * 3) * 100) = 116.45856$$

$$Yao = 0.44588 - (\cos(0.22157 * 3) * 100) = -99.54739$$

$$Cao = \sqrt{116.45856^2 + (-99.54739 + 100)^2} = 116.45944$$

$$Lsao = 116.45944 * 115.30 / 115.29931 = 116.46014$$

$$DEFao = \arctan((-99.54739 + 100) / 116.45856) = 0.22268^\circ \text{ or } 0^\circ 13' 22''$$



METHOD NO. 2
Not to scale

FORMULAS

$$C = Lsl - (0.00034 * a^2 * (Lsl / 100)^5); \quad Lsl = POS - TS$$

$$DEF = (a * Lsl^2) / 60000$$

$$Xb = C * \cos(DEF)$$

$$Yb = C * \sin(DEF)$$

$$Xbi = Xb - (\sin(DEF * 3) * W)$$

$$Ybi = Yb + (\cos(DEF * 3) * W) - W$$

$$Xbo = Xb + (\sin(DEF * 3) * W)$$

$$Ybo = Yb + W - (\cos(DEF * 3) * W)$$

$$Cbi = \sqrt{Xbi^2 + Ybi^2}$$

$$Lsbi = Cbi * Lsl / C$$

$$DEFbi = \arctan(Ybi / Xbi)$$

$$Cbo = \sqrt{Xbo^2 + Ybo^2}$$

$$Lsbo = Cbo * Lsl / C$$

$$DEFbo = \arctan(Ybo / Xbo)$$

EXAMPLE

Given: $a = 1$ $TS = 2180+84.70$ (From SC-1) $POS = 2182+50.00$ $W = 100'$

$$Lsl = 2182+50.00 - 2180+84.70 = 165.30'$$

$$C = 165.30 - (0.00034 * 1^2 * (165.30 / 100)^5) = 165.29580$$

$$DEF = (1 * 165.30^2) / 60000 = 0.45540^\circ \text{ or } 0^\circ 27' 19''$$

$$Xb = 165.29580 * \cos(0.45540) = 165.29058$$

$$Yb = 165.29580 * \sin(0.45540) = 1.31380$$

$$Xbi = 165.29058 - (\sin(0.45540 * 3) * 100) = 162.90634$$

$$Ybi = 1.31380 + (\cos(0.45540 * 3) * 100) - 100 = 1.28537$$

$$Cbi = \sqrt{162.90634^2 + 1.28537^2} = 162.91141$$

$$Lsbi = 162.91141 * 165.30 / 165.29580 = 162.91555$$

$$DEFbi = \arctan(1.28537 / 162.90634) = 0.45207^\circ \text{ or } 0^\circ 27' 07''$$

$$Xbo = 165.29058 + (\sin(0.45540 * 3) * 100) = 167.67482$$

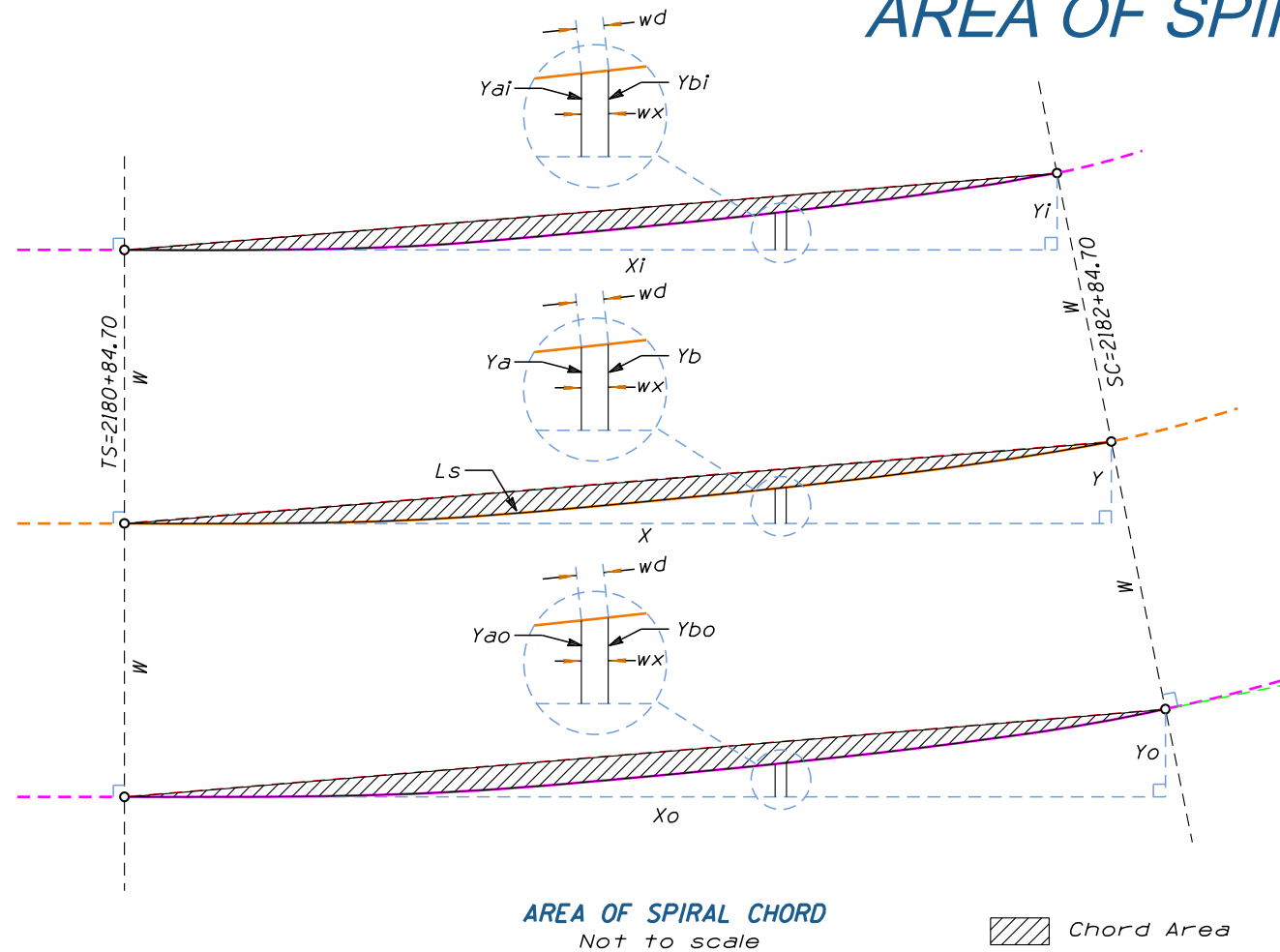
$$Ybo = 1.31380 + 100 - (\cos(0.45540 * 3) * 100) = 1.34223$$

$$Cbo = \sqrt{167.67482^2 + 1.34223^2} = 167.68019$$

$$Lsbo = 167.67882 * 165.30 / 165.29580 = 167.68308$$

$$DEFbo = \arctan(1.34223 / 167.67482) = 0.45864^\circ \text{ or } 0^\circ 27' 31''$$

AREA OF SPIRAL CHORD



FORMULAS

Given: $a = 1$, $L_s = 200$ (From SC-1); $W = 100'$,
 $L_{s1} = 196.50939$, $X_i = 196.48563$, $Y_i = 2.26601$ (From SC-2)
 $wd = 1$ (Interval distance) (Area precision is based upon wd)
 wx (Segment width)

$$\text{Chord Area} = (X_i * Y_i / 2) - \sum_{L_{s1}=wd}^{L_s} (Y_{ai} + Y_{bi}) / 2 * wx$$

Step 1:

$$X_i * Y_i / 2 = 196.48563 * 2.26601 / 2 = \underline{222.61920\#}$$

Step 2

$$\text{Summation: } \sum_{L_{s1}=wd}^{L_s} (Y_{ai} + Y_{bi}) / 2 * wx$$

Start $L_{s1} = wd$:

$$\text{DefY}_a = a * L_{s1}^2 / 60000; \text{DefY}_b = a * (L_{s1} + wd)^2 / 60000$$

$$Y_{ai} = (L_{s1} - (0.00034 * a^2 * (L_{s1} / 100)^5)) * \sin(\text{DefY}_a) + ((\cos(\text{DefY}_a * 3) * W) - W)$$

$$Y_{bi} = ((L_{s1} + wd) - (0.00034 * a^2 * ((L_{s1} + wd) / 100)^5)) * \sin(\text{DefY}_b) + ((\cos(\text{DefY}_b * 3) * W) - W)$$

$$wx = \sqrt{(wd * L_{s1} / L_s)^2 - (Y_{bi} - Y_{ai})^2}$$

$$\text{Segment Area} = (Y_{ai} + Y_{bi}) / 2 * wx$$

$$L_{s1} = L_{s1} + wd$$

Repeat until $L_{s1} = L_s$

$$\text{Area below spiral curve} = \underline{111.88952\#}$$

Step 3:

$$\text{Chord Area} = 222.61920\# - 111.88952\# = \underline{110.72968\#}$$

INSIDE SPIRAL

FORMULAS

Given: $a = 1$, $L_s = 200$, $X = 199.97558$, $Y = 2.32693$ (From SC-1)
 $wd = 1$ (Interval distance) (Area precision is based upon the value of wd)
 wx (Segment width)

$$\text{Chord Area} = (X * Y / 2) - \sum_{L_{s1}=wd}^{L_s} (Y_a + Y_b) / 2 * wx$$

Step 1:

$$X * Y / 2 = 199.97558 * 2.32693 / 2 = \underline{232.66459\#}$$

Step 2

$$\text{Summation: } \sum_{L_{s1}=wd}^{L_s} (Y_a + Y_b) / 2 * wx$$

Start $L_{s1} = wd$:

$$Y_a = (L_{s1} - (0.00034 * a^2 * (L_{s1} / 100)^5)) * \sin(a * L_{s1}^2 / 60000)$$

$$Y_b = ((L_{s1} + wd) - (0.00034 * a^2 * ((L_{s1} + wd) / 100)^5)) * \sin(a * (L_{s1} + wd)^2 / 60000)$$

$$wx = \sqrt{wd^2 - (Y_b - Y_a)^2}$$

$$\text{Segment Area} = (Y_a + Y_b) / 2 * wx$$

$$L_{s1} = L_{s1} + wd$$

Repeat until $L_{s1} = L_s$

$$\text{Area below spiral curve} = \underline{116.31862\#}$$

Step 3

$$\text{Chord Area} = 232.66459\# - 116.31862\# = \underline{116.34597\#}$$

CENTERLINE SPIRAL

FORMULAS

Given: $a = 1$, $L_s = 200$ (From SC-1); $W = 100'$,
 $L_{s0} = 203.49061$, $X_o = 203.46553$, $Y_o = 2.38785$ (From SC-2)
 $wd = 1$ (Interval distance) (Area precision is based upon wd)
 wx (Segment width)

$$\text{Chord Area} = (X_o * Y_o / 2) - \sum_{L_{s1}=wd}^{L_s} (Y_{ao} + Y_{bo}) / 2 * wx$$

Step 1:

$$X_o * Y_o / 2 = 203.46553 * 2.38785 / 2 = \underline{242.92258\#}$$

Step 2

$$\text{Summation: } \sum_{L_{s1}=wd}^{L_s} (Y_{ao} + Y_{bo}) / 2 * wx$$

Start $L_{s1} = wd$:

$$\text{DefY}_a = a * L_{s1}^2 / 60000; \text{DefY}_b = a * (L_{s1} + wd)^2 / 60000$$

$$Y_{ao} = (L_{s1} - (0.00034 * a^2 * (L_{s1} / 100)^5)) * \sin(\text{DefY}_a) + (W - (\cos(\text{DefY}_a * 3) * W))$$

$$Y_{bo} = ((L_{s1} + wd) - (0.00034 * a^2 * ((L_{s1} + wd) / 100)^5)) * \sin(\text{DefY}_b) + (W - (\cos(\text{DefY}_b * 3) * W))$$

$$wx = \sqrt{(wd * L_{s0} / L_s)^2 - (Y_{bo} - Y_{ao})^2}$$

$$\text{Segment Area} = (Y_{ao} + Y_{bo}) / 2 * wx$$

$$L_{s1} = L_{s1} + wd$$

Repeat until $L_{s1} = L_s$

$$\text{Area below spiral curve} = \underline{120.82001\#}$$

Step 3:

$$\text{Chord Area} = 242.92258\# - 120.82001\# = \underline{122.10257\#}$$

OUTSIDE SPIRAL

AREA OF SPIRAL CHORD

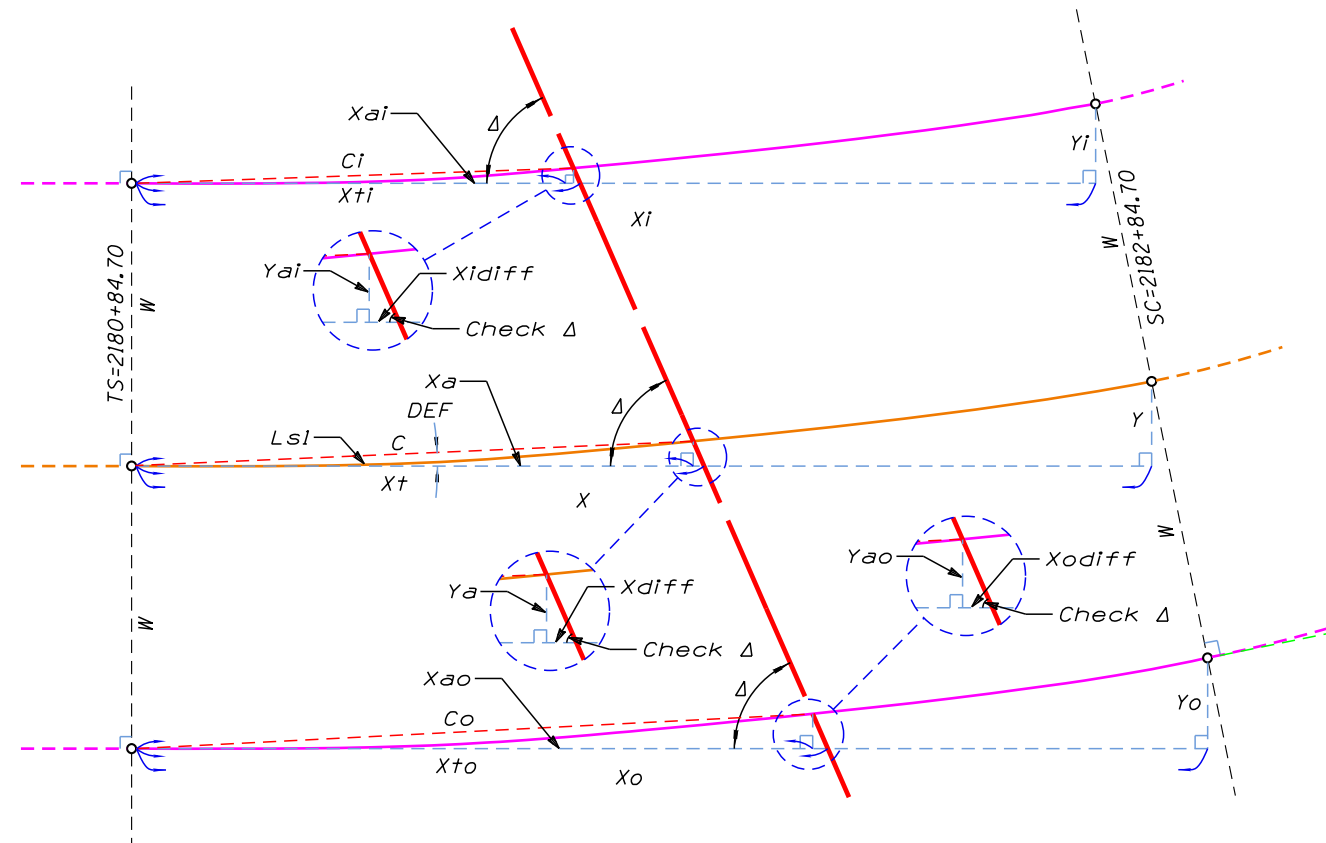
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Date	Jun 2011	Project Sur.	Jim Crume
Project No.	ADOT Spiral	Project Manager	Jim Crume

SC-4
Sht: 4 of 7

SPIRAL CURVE - LINE INTERSECTION



INTERSECTION
Not to scale

FORMULAS

Given: $a = 1.00$, $\Delta = 66^\circ 20' 20''$, $X_{ti} = 76.43$, $W = 100.00$

$$\begin{aligned} C &= L_{sl} - (0.00034 * a^2 * (L_{sl} / 100)^5) \\ DEF &= (a * L_{sl}^2) / 60000 \\ X_a &= C * \cos(DEF) \\ Y_a &= C * \sin(DEF) \\ X_{ai} &= X_a - (\sin(DEF * 3) * W) \\ Y_{ai} &= Y_a + (\cos(DEF * 3) * W) - W \\ X_{diff} &= X_{ti} - X_{ai} \\ \text{Check } \Delta &= \text{ArcTan}(Y_{ai} / X_{diff}) \end{aligned}$$

Starting value needs to be larger than X_{ti} in order to find a solution.

Begin Iteration

Starting values: $L_{sl} = 100'$, Tolerance = 0.0001

$$\begin{aligned} C &= 100 - (0.00034 * 1^2 * (100 / 100)^5) = 99.99966 \\ DEF &= (1 * 100^2) / 60000 = 0.166667^\circ \text{ or } 0^\circ 10' 00'' \\ X_a &= 99.99966 * \cos(0.166667) = 99.99924 \\ Y_a &= 99.99966 * \sin(0.166667) = 0.29089 \\ X_{ai} &= 99.99924 - (\sin(0.166667 * 3) * 100.00) = 99.12658 \\ Y_{ai} &= 0.29089 + (\cos(0.166667 * 3) * 100.00) - 100.00 = 0.28708 \\ X_{diff} &= 76.43 - 99.12658 = -22.69658 \\ \text{Check } \Delta &= \text{ArcTan}(0.28708 / -22.69658) = -0.72467^\circ \text{ or } -0^\circ 43' 28.8'' \end{aligned}$$

Note:
Rounding error will occur based upon the number of decimal places used.

Δ - Check $\Delta = 66.33889 - 0.72467 = 65.61422$ (If \leq Tolerance then solution found) If no solution found then $L_{sl} = L_{sl} - \text{Tolerance}$. Repeat iteration with new L_{sl} until solution is found. The solution is best found by utilizing a computer program to run the iterations.

INSIDE SPIRAL INTERSECTION

FORMULAS

Given: $a = 1.00$, $\Delta = 66^\circ 20' 20''$, $X_t = 120.25$

$$\begin{aligned} C &= L_{sl} - (0.00034 * a^2 * (L_{sl} / 100)^5) \\ DEF &= (a * L_{sl}^2) / 60000 \\ X_a &= C * \cos(DEF) \\ Y_a &= C * \sin(DEF) \\ X_{diff} &= X_t - X_a \\ \text{Check } \Delta &= \text{ArcTan}(Y_a / X_{diff}) \end{aligned}$$

Begin Iteration

Starting values: $L_{sl} = 200'$, Tolerance = 0.0001

$$\begin{aligned} C &= 200 - (0.00034 * 1^2 * (200 / 100)^5) = 199.98912 \\ DEF &= (1 * 200^2) / 60000 = 0.666667^\circ \text{ or } 0^\circ 40' 00'' \\ X_a &= 199.98912 * \cos(0.666667) = 199.97558 \\ Y_a &= 199.98912 * \sin(0.666667) = 2.32693 \\ X_{diff} &= 120.25 - 199.98912 = -79.73912 \\ \text{Check } \Delta &= \text{ArcTan}(2.32693 / -79.73912) = -1.671519^\circ \text{ or } -1^\circ 40' 17.5'' \end{aligned}$$

Δ - Check $\Delta = 66.33889 - 1.671519 = 64.667371$ (If \leq Tolerance then solution found) If no solution found then $L_{sl} = L_{sl} - \text{Tolerance}$. Repeat iteration with new L_{sl} until solution found.

The solution is best found by utilizing a computer program to run the iterations.

CENTERLINE SPIRAL INTERSECTION

Solution

$$\begin{aligned} L_{sl} &= 120.03149 \\ C &= 120.03064 \\ DEF &= 0.240126^\circ \\ X_a &= 120.02959 \\ Y_a &= 0.50305 \\ X_{diff} &= 0.22041 \\ \text{Check } \Delta &= 66.338883^\circ \end{aligned}$$

Note:
Rounding error will occur based upon the number of decimal places used.

FORMULAS

Given: $a = 1.00$, $\Delta = 66^\circ 20' 20''$, $X_{to} = 164.07$, $W = 100.00$

$$\begin{aligned} C &= L_{sl} - (0.00034 * a^2 * (L_{sl} / 100)^5) \\ DEF &= (a * L_{sl}^2) / 60000 \\ X_a &= C * \cos(DEF) \\ Y_a &= C * \sin(DEF) \\ X_{ao} &= X_a + (\sin(DEF * 3) * W) \\ Y_{ao} &= Y_a + W - (\cos(DEF * 3) * W) \\ X_{diff} &= X_{to} - X_{ao} \\ \text{Check } \Delta &= \text{ArcTan}(Y_{ao} / X_{diff}) \end{aligned}$$

Begin Iteration

Starting values: $L_{sl} = 180'$, Tolerance = 0.0001

$$\begin{aligned} C &= 180 - (0.00034 * 1^2 * (180 / 100)^5) = 179.99358 \\ DEF &= (1 * 180^2) / 60000 = 0.54000^\circ \text{ or } 0^\circ 32' 24'' \\ X_a &= 179.99358 * \cos(0.54000) = 179.98559 \\ Y_a &= 179.99358 * \sin(0.54000) = 1.69637 \\ X_{ao} &= 179.98559 + (\sin(0.54000 * 3) * 100.00) = 182.81264 \\ Y_{ao} &= 1.69637 + 100.00 - (\cos(0.54000 * 3) * 100.00) = 1.73634 \\ X_{diff} &= 164.07 - 182.81264 = -18.74264 \\ \text{Check } \Delta &= \text{ArcTan}(1.73634 / -18.74264) = -5.29284^\circ \text{ or } -5^\circ 17' 34.2'' \end{aligned}$$

Starting value needs to be larger than X_{to} in order to find a solution.

Note:
Rounding error will occur based upon the number of decimal places used.

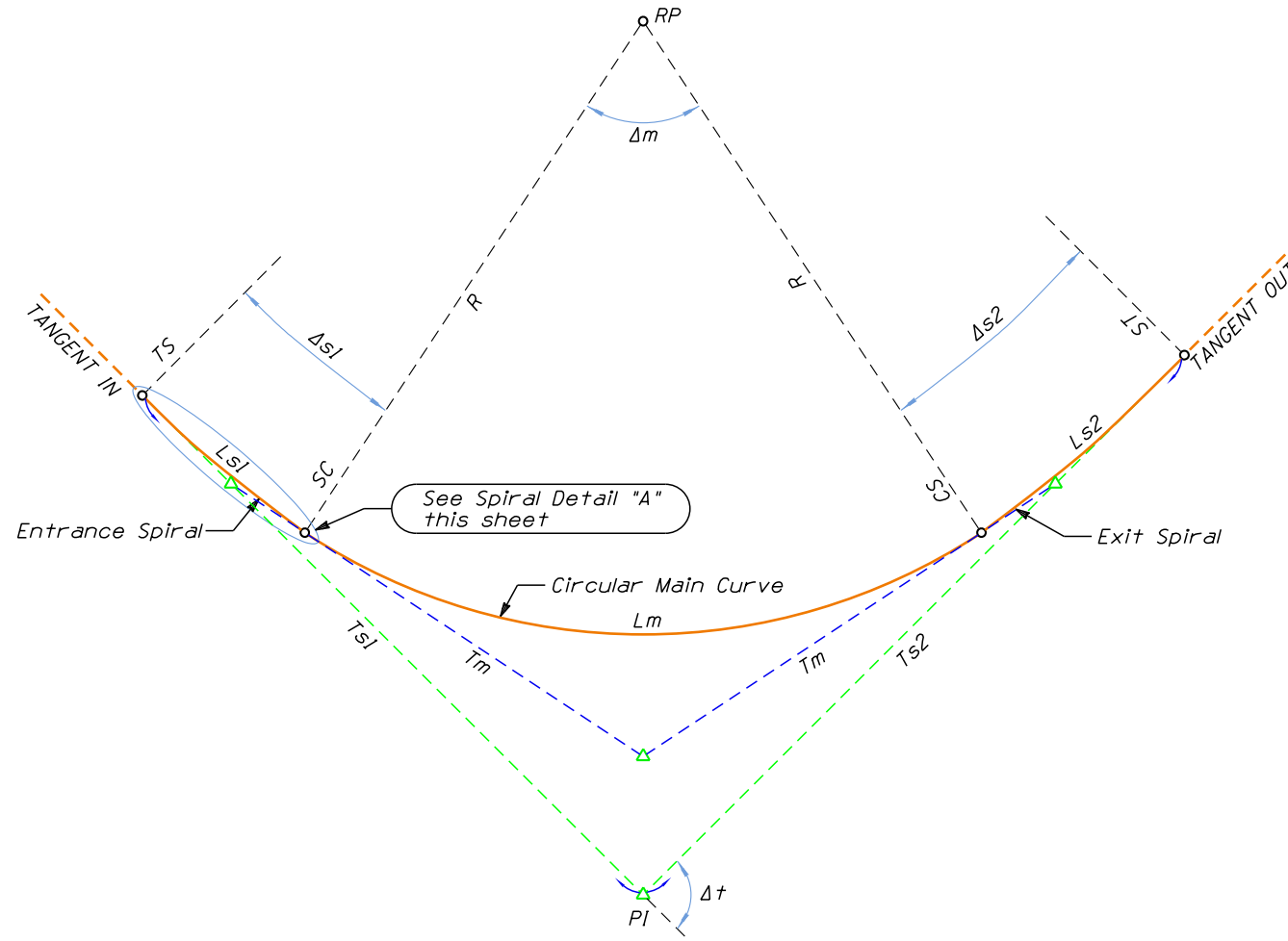
Δ - Check $\Delta = 66.33889 - 5.29284 = 61.04605$ (If \leq Tolerance then solution found) If no solution found then $L_{sl} = L_{sl} - \text{Tolerance}$. Repeat iteration with new L_{sl} until solution is found. The solution is best found by utilizing a computer program to run the iterations.

OUTSIDE SPIRAL INTERSECTION

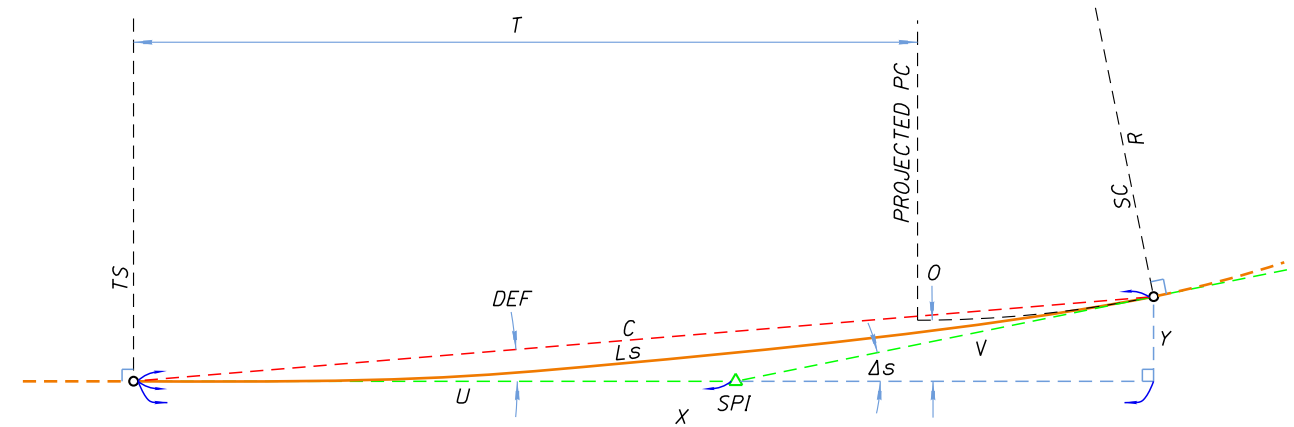
Solution

$$\begin{aligned} L_{sl} &= 76.88876 \\ C &= 76.88867 \\ DEF &= 0.09853^\circ \\ X_a &= 76.88855 \\ Y_a &= 0.13223 \\ X_{ai} &= 76.37264 \\ Y_{ai} &= 0.13089 \\ X_{diff} &= 0.05736 \\ \text{Check } \Delta &= 66.3389^\circ \end{aligned}$$

FULL TRANSITION UNEQUAL SPIRAL CURVES



TYPICAL HIGHWAY SPIRAL-MAIN-SPIRAL CURVE
Not to scale



SPIRAL DETAIL "A"
Not to scale

Highway Spiral Curve where $\Delta s < 16^\circ$

FORMULAS

See SC-1 for spiral formulas

$$\begin{aligned}\Delta m &= \Delta t - \Delta s1 - \Delta s2 \\ Xa &= \cos(\Delta s1) * (V1 + Tm) \\ Ya &= \sin(\Delta s1) * (V1 + Tm) \\ Xb &= \cos(\Delta m + \Delta s1) * (V2 + Tm) \\ Yb &= \sin(\Delta m + \Delta s1) * (V2 + Tm) \\ Xc &= (Ya + Yb) / \tan(\Delta t) \\ Ts1 &= Xa + Xb - Xc + U1 \\ Ts2 &= \sqrt{Xc^2 + (Ya + Yb)^2} + U2\end{aligned}$$

EXAMPLE

Given: $\Delta t = 36^\circ 29' 16''$; $D = 2^\circ 00' 00''$; $Ls1 = 200'$; $Ls2 = 300'$; $TS(Sta) = 2180+84.70$

Entrance Spiral (Ls1)

$$\begin{aligned}a &= (2.0000^\circ * 100) / 200 = 1.00 \\ O &= 0.0727 * 1 * ((200 / 100)^3) = 0.58160 \\ T &= (200 / 2) - (0.000127 * 1^2 * (200 / 100)^5) = 99.99594 \\ C &= 200 - (0.00034 * 1^2 * (200 / 100)^5) = 199.98912 \\ DEF &= (1 * 200^2) / 60000 = 0.666667^\circ \text{ or } 0^\circ 40' 00'' \\ \Delta s &= 0.005 * 2.0000^\circ * 200 = 2.0000^\circ \text{ or } 2^\circ 00' 00'' \\ U &= 199.98912 * \sin(2.0000^\circ * 2 / 3) / \sin(2.0000^\circ) = 133.34112 \\ V &= 199.98912 * \sin(2.0000^\circ * 1 / 3) / \sin(2.0000^\circ) = 66.67508 \\ X &= 199.98912 * \cos(0.666667^\circ) = 199.97558 \\ Y &= 199.98912 * \sin(0.666667^\circ) = 2.32693\end{aligned}$$

Main Curve

$$\begin{aligned}R &= 5729.57795 / 2.0000^\circ = 2864.78898 \\ \Delta m &= 36^\circ 29' 16'' - 2^\circ 00' 00'' - 3^\circ 00' 00'' = 31^\circ 29' 16'' \text{ or } 31.48778^\circ \\ Lm &= (31.48778^\circ * 2864.78898 * 3.141592654) / 180 = 1574.38900 \\ Tm &= 2864.78898 * \tan(31.48778 / 2) = 807.62426\end{aligned}$$

$$\begin{aligned}SC(Sta) &= 2180+84.70 + 200 = 2182+84.70 \\ CS(Sta) &= 2182+84.70 + 1574.39 = 2198+59.09 \\ ST(Sta) &= 2198+59.09 + 300 = 2201+59.09 \\ PI(Sta) &= 2180+84.70 + 1045.74 = 2191+30.44\end{aligned}$$

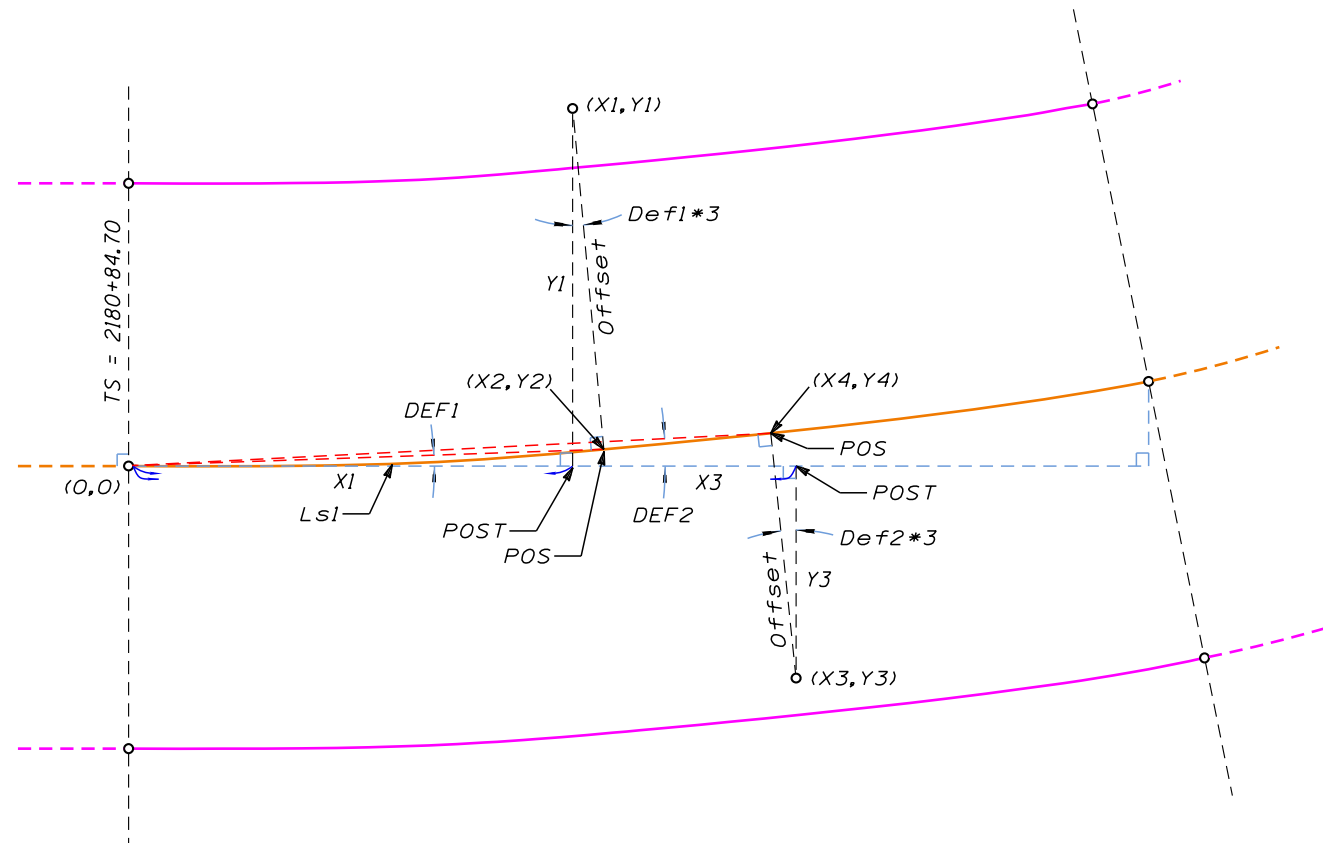
Exit Spiral (Ls2)

$$\begin{aligned}a &= (2.0000^\circ * 100) / 300 = 0.66667 \\ O &= 0.0727 * 0.66667 * ((300 / 100)^3) = 1.30860 \\ T &= (300 / 2) - (0.000127 * 0.66667^2 * (300 / 100)^5) = 149.98628 \\ C &= 300 - (0.00034 * 0.66667^2 * (300 / 100)^5) = 299.96328 \\ DEF &= (0.66667 * 300^2) / 60000 = 1.000^\circ \text{ or } 01^\circ 00' 00'' \\ \Delta s &= 0.005 * 2.0000^\circ * 300 = 3.000^\circ \text{ or } 3^\circ 00' 00'' \\ U &= 299.96328 * \sin(3.000^\circ * 2 / 3) / \sin(3.000^\circ) = 200.02630 \\ V &= 299.96328 * \sin(3.000^\circ * 1 / 3) / \sin(3.000^\circ) = 100.02838 \\ X &= 299.96328 * \cos(1.000^\circ) = 299.91759 \\ Y &= 299.96328 * \sin(1.000^\circ) = 5.23508\end{aligned}$$

Tangents

$$\begin{aligned}Xa &= \cos(2.000) * (66.67508 + 807.62426) = 873.76674 \\ Ya &= \sin(2.000) * (66.67508 + 807.62426) = 30.51261 \\ Xb &= \cos(31.48778 + 2.000) * (100.02838 + 807.62426) = 756.98550 \\ Yb &= \sin(31.48778 + 2.000) * (100.02838 + 807.62426) = 500.80562 \\ Xc &= (30.51261 + 500.80562) / \tan(36.48778) = 718.35575 \\ Ts1 &= 873.76674 + 756.98550 - 718.35575 + 133.34112 = 1045.73711 \\ Ts2 &= \sqrt{718.35575^2 + (30.51261 + 500.80562)^2} + 200.02630 = 1093.52171\end{aligned}$$

SPIRAL CURVE - STATION - OFFSET



STATION - OFFSET
Not to scale

FORMULAS

Given: $TS(Sta) = 2180+84.70$; $X = 0$, $Y = 0$; $X1 = 94.500$, $Y1 = 110.400$;
 $Ls = 200'$; $D = 2^\circ 00' 00''$; $a = 1.000$

P.O.S.T.

$Sta = TS + X1$; $Offset = Y1$

P.O.S.

Begin Iteration

Starting values: $Ls1 = 200'$, $Tolerance = 0.0001$

$$C = Ls1 - (0.00034 * a^2 * (Ls1 / 100)^5)$$

$$DEF1 = (a * Ls1^2) / 60000$$

$$X2 = C * \cos(DEF1)$$

$$Y2 = C * \sin(DEF1)$$

$$Xdifff = X2 - X1$$

$$Ydifff = Y2 - Y1$$

$$Check \Delta = \text{ArcTan}(Xdifff / Ydifff)$$

If $Def1 * 3 = Check \Delta$ then solution found

If no solution found then $Ls1 = Ls1 - Tolerance$.

Repeat iteration with new $Ls1$ until solution found.

$$Sta = TS + Ls1$$

$$Offset = \sqrt{Xdifff^2 + Ydifff^2}$$

Solution

$$2180+84.70 + 94.50 =$$

$$2181+79.20 \text{ (P.O.S.T.)}$$

$$110.40 \text{ LT}$$

$$Ls1 = 95.37476 \quad C = 95.37479$$

$$DEF1 = 0.15161^\circ \text{ or } 00^\circ 09' 05.8''$$

$$X2 = 95.37446$$

$$Y2 = 0.25236$$

$$Xdifff = 0.87446$$

$$Ydifff = -110.14764$$

$$0.45482 - 0.45486 = -0.00004$$

less than 0.0001 (Tolerance)

Solution found

$$2180+84.70 + 95.37 =$$

$$2181+80.07 \text{ (P.O.S.)}$$

$$\sqrt{0.87446^2 + (-110.14764)^2}$$

$$= 110.15 \text{ (Offset LT)}$$

OFFSET - LEFT SIDE

FORMULAS

Given: $TS(Sta) = 2180+84.70$; $X = 0$, $Y = 0$; $X1 = 94.500$, $Y1 = 110.400$; $X3 = 125.400$, $Y3 = -80.500$
 $Ls = 200'$; $D = 2^\circ 00' 00''$; $a = 1.000$

P.O.S.T.

$Sta = TS + X1$ and $TS + X3$; $Offset = Y1$ and $Y3$

P.O.S.

Begin Iteration

Starting values: $Ls1 = 200'$, $Tolerance = 0.0001$

$$C = Ls1 - (0.00034 * a^2 * (Ls1 / 100)^5)$$

$$DEF1 \text{ or } DEF2 = (a * Ls1^2) / 60000$$

$$X2 \text{ or } X4 = C * \cos(DEF1 \text{ or } DEF2)$$

$$Y2 \text{ or } Y4 = C * \sin(DEF1 \text{ or } DEF2)$$

$$Xdifff = X2 - X1 \text{ or } X4 - X3$$

$$Ydifff = Y2 - Y1 \text{ or } Y4 - Y3$$

$$Check \Delta = \text{ArcTan}(Xdifff / Ydifff)$$

If $Def1 * 3$ or $Def2 * 3 = Check \Delta$ then solution found

If no solution found then $Ls1 = Ls1 - Tolerance$.

Repeat iteration with new $Ls1$ until solution found.

The solution is best found by utilizing a computer program to run the iterations.

$$Sta = TS + Ls1; \quad Offset = \sqrt{Xdifff^2 + Ydifff^2}$$

CENTERLINE SPIRAL STATION-OFFSET

FORMULAS

Given: $TS(Sta) = 2180+84.70$; $X = 0$, $Y = 0$; $X3 = 125.400$, $Y3 = -80.500$;
 $Ls = 200'$; $D = 2^\circ 00' 00''$; $a = 1.000$

P.O.S.T.

$Sta = TS + X3$; $Offset = Y3$

P.O.S.

Begin Iteration

Starting values: $Ls1 = 200'$, $Tolerance = 0.0001$

$$C = Ls1 - (0.00034 * a^2 * (Ls1 / 100)^5)$$

$$DEF2 = (a * Ls1^2) / 60000$$

$$X4 = C * \cos(DEF2)$$

$$Y4 = C * \sin(DEF2)$$

$$Xdifff = X4 - X3$$

$$Ydifff = Y4 - Y3$$

$$Check \Delta = \text{ArcTan}(Xdifff / Ydifff)$$

If $Def2 * 3 = Check \Delta$ then solution found

If no solution found then $Ls1 = Ls1 - Tolerance$.

Repeat iteration with new $Ls1$ until solution found.

$$Sta = TS + Ls1$$

$$Offset = \sqrt{Xdifff^2 + Ydifff^2}$$

Solution

$$2180+84.70 + 125.400 =$$

$$2182+10.10 \text{ (P.O.S.T.)}$$

$$80.50 \text{ RT}$$

$$Ls1 = 124.31152 \quad C = 124.30807$$

$$DEF2 = 0.25754^\circ \text{ or } 00^\circ 15' 27.1''$$

$$X4 = 124.30682$$

$$Y4 = 0.55877$$

$$Xdifff = -1.09318$$

$$Ydifff = 81.05877$$

$$0.77263 - 0.77266 = -0.00004$$

less than 0.0001 (Tolerance)

Solution found

$$2180+84.70 + 124.31 =$$

$$2182+09.01 \text{ (P.O.S.)}$$

$$\sqrt{(-1.09318)^2 + 81.05877^2}$$

$$= 81.07 \text{ (Offset RT)}$$

OFFSET - RIGHT SIDE

SPIRAL CURVE -
STATION - OFFSET

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Date
Jun 2011
Project Sur.
Jim Crume

Project No.
A00T Spiral
Project Manager
Jim Crume

SC-7
Sht: 7 of 7